Center dominance, Casimir scaling, and confinement in lattice gauge theory [*]

L. Del Debbio [1], **M. Faber** [2], **J. Greensite** [3], **Š. Olejník** [4]

[1] Dept. of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK
[2] Institut für Kernphysik, Technische Universität Wien, A–1040 Vienna, Austria
[3] The Niels Bohr Institute, DK–2100 Copenhagen Ø, Denmark
[4] Institute of Physics, Slovak Academy of Sciences, SK–842 28 Bratislava, Slovakia

Abstract: We present numerical evidence that supports the theory of quark confinement based on center vortex condensation. We introduce a special gauge ("maximal center gauge") and center projection, suitable for identification of center vortices. Main focus is then put on the connection of vortices in center projection to "confiners" in full, unprojected gauge-field configurations. Topics briefly discussed include: the relation between vortices and monopoles, first results for SU(3), and the problem of Casimir scaling.

1 Introduction

The most popular model of colour confinement in QCD relies on the idea of "dual superconductivity", due to 't Hooft and Mandelstam. A realization of the idea is the abelian-projection theory of 't Hooft [1]: he suggested to fix to an "abelian projection" gauge, reducing the SU(N) gauge symmetry to U(1)$^{N-1}$, and identifying abelian gauge fields (with respect to the residual symmetry) and magnetic monopoles. Abelian electric charges then become confined due to monopole condensation. In 1987, Kronfeld et al. [2] suggested testing 't Hooft's theory in lattice simulations, in a special gauge that makes SU(N) link variables as diagonal as possible. If one computes various physical observables using the diagonal parts of the links only, one observes "abelian" dominance [3]: the expectation values of the physical quantities in the full non-abelian theory (often) coincide with the ones in the abelian theory obtained by the abelian projection in the maximal abelian gauge. Much evidence has been obtained for the abelian-projection picture and the model of dual superconductivity, but there remain problems to be solved. We have underlined its inability to explain approximate Casimir scaling of the linear potential between higher-representation colour sources at intermediate length scales [4, 5].

Another picture of confinement was quite popular before the advent of dual superconductivity, namely the Z_N vortex condensation theory, proposed, in various forms, by many authors [6]. According to this model, the QCD vacuum is filled with vortices, having the topology of tubes (in 3 Euclidean dimensions) or surfaces (in 4 dimensions) of finite thickness, which carry magnetic flux quantized in terms of elements of the center of the gauge group. Center vortices are assumed to condense in the QCD vacuum. The area-law fall-off of large Wilson loops comes from fluctuations in the number of center vortices linked to the loops.
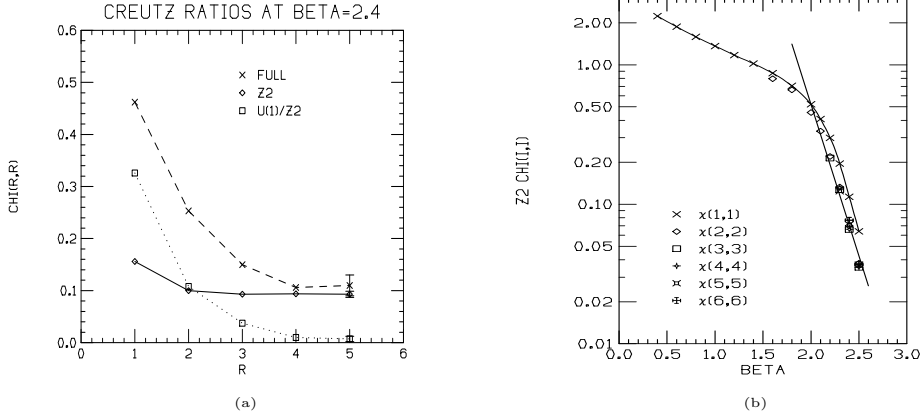
Figure 1: Creutz ratios: (a) vs. R for full, center-projected, and U(1)/Z_2-projected lattice configurations at $\beta = 2.4$ (IMCG), (b) vs. β from center-projected configurations (DMCG). The straight line is the asymptotic freedom prediction: $\sigma a^2 = (\sigma/\Lambda^2)\left(6\pi^2\beta/11\right)^{102/121}\exp\left[-6\pi^2\beta/11\right]$.

The Z_N vortex condensation theory apparently suffers from the same "Casimir-scaling disease" as the abelian projection does: there seems no way of accommodating the existence of a linear potential between adjoint sources to the idea of vortices dominating the QCD vacuum. There exists a simple solution to this controversy [7], and we will discuss it at the end of this paper.

With perhaps only one exception [8], the ideas behind the vortex-condensation picture have not in the past been subjected to lattice tests. The aim of our investigation is to study the vortex theory in numerical Monte Carlo simulations, by methods and approaches inspired to some extent by earlier work of many authors in the abelian projection theory.

2   Maximal center gauge, projection and dominance

The maximal abelian gauge underscores the role of the largest abelian subgroup of the gauge group. In much the same way, one can choose a gauge condition in which the gauge group *center* is given prominent importance. In SU(2) lattice gauge theory we proposed [5, 9] to fix to *maximal center gauge* (MCG) by making link variables U as close as possible to its center elements $\pm I$. There are many (in fact infinitely many) ways how to do it; we implemented two simple choices:

1. The *indirect maximal center gauge* (IMCG) [5, 9–11]: We first fix to maximal abelian gauge (MAG) in the usual way, by maximizing the quantity

$$\sum_x \sum_\mu \mathrm{Tr}\,[U_\mu(x)\sigma_3 U_\mu^\dagger(x)\sigma_3], \tag{1}$$

then extract from $U_\mu(x)$ their diagonal parts $A_\mu(x) = \exp[i\theta_\mu(x)\sigma_3]$, and use the remnant U(1) symmetry to bring $A_\mu(x)$ as close as possible to the center elements by maximizing

$$\sum_x \sum_\mu \cos^2\theta_\mu(x). \tag{2}$$

2. The *direct maximal center gauge* (DMCG) [10, 12]: To fix this gauge one looks directly for the maximum of

$$\sum_x \sum_\mu \mathrm{Tr}\,[U_\mu(x)]\mathrm{Tr}\,[U_\mu^\dagger(x)]. \tag{3}$$

In both cases, after gauge fixing one is left with the remnant Z_2 gauge symmetry.[†]

[†]Qualitatively, the same physical results are obtained in both gauges (cf. [9, 12]).
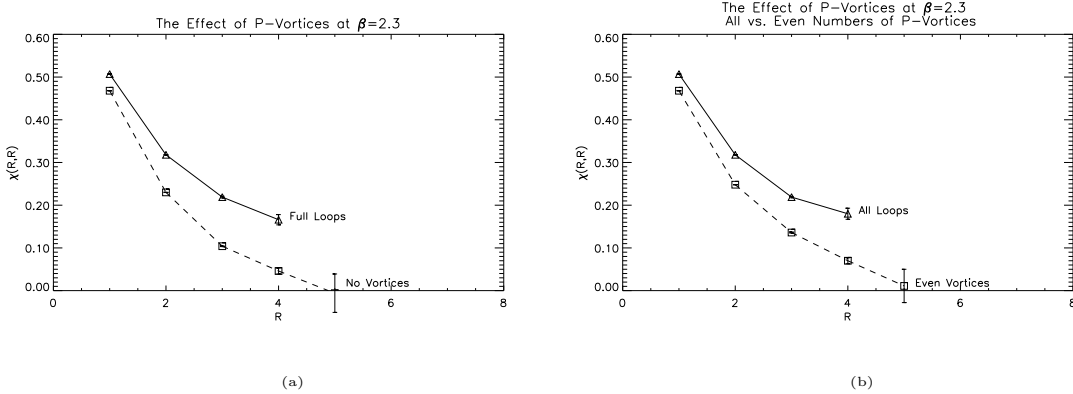
Figure 2: Creutz ratios extracted from loops with (a) no P-vortices, (b) even number of P-vortices piercing the loop, compared with the usual Creutz ratios at $\beta = 2.3$.

The next step is *center projection*, i.e. replacing full link matrices U (in a particular MCG) by center elements Z, which are defined to be

$$Z \equiv \text{sign}\left(\cos(\theta)\right) I \quad \text{(in IMCG)} \qquad \text{or} \qquad Z \equiv \text{sign}\left(\text{Tr}\left(U\right)\right) I \quad \text{(in DMCG)}, \tag{4}$$

and to compute various physical quantities of interest, e.g. Wilson loops and Creutz ratios, using the Z links.

Figure 1a compares Creutz ratios $\chi(R,R)$ at $\beta = 2.4$ computed from full lattice configurations and from center-projected configurations. We clearly see *center dominance*: Creutz ratios computed from Z links agree with full Creutz ratios at large enough distances, the asymptotic values of the string tension almost coincide. On the contrary, Creutz ratios computed from links with the Z variable factored out (dotted line in Figure 1a) show no string tension at all. Another interesting observation is that the Creutz ratios computed from center-projected configurations almost do not depend on R; center projection removes Coulombic contributions.

In Figure 1b we plot Creutz ratios vs. β, extracted from center-projected configurations in DMCG. The straight line is the asymptotic freedom prediction with the value of $\sqrt{\sigma}/\lambda = 58$, which very well agrees with "state-of-the-art" asymptotic string tension computations [13].

Our data show that the Z center variables are crucial parts of the U links in MCG, in particular they carry most of the information on the string tension. This phenomenon of *center dominance* gives rise to a whole series of questions on the role and nature of center vortex configurations in the QCD vacuum. We will list a few of those questions here and sketch our tentative answers.

3 Questions and answers

3.1 Vortices and confinement?

Question 1: Has center dominance anything to do with confinement? What, if any, is the relation of Z_2 vortices seen after center projection to "confiners" in full, unprojected configurations?

To answer the question, we first introduce the notion of a *P(rojection)-vortex*. The excitations of a Z_2 lattice gauge theory with non-zero action are "thin" vortices, having the topology of a surface, one lattice spacing thick. We will call such vortices in center projected Z-link configurations P-vortices. A plaquette is pierced by a P-vortex if, after maximal center gauge fixing and center projection, the corresponding projected plaquette has the value of -1.

However, we want to emphasize that center projection, and abelian projection as well, represents an uncontrollable truncation of full lattice configurations. Therefore we will not base our
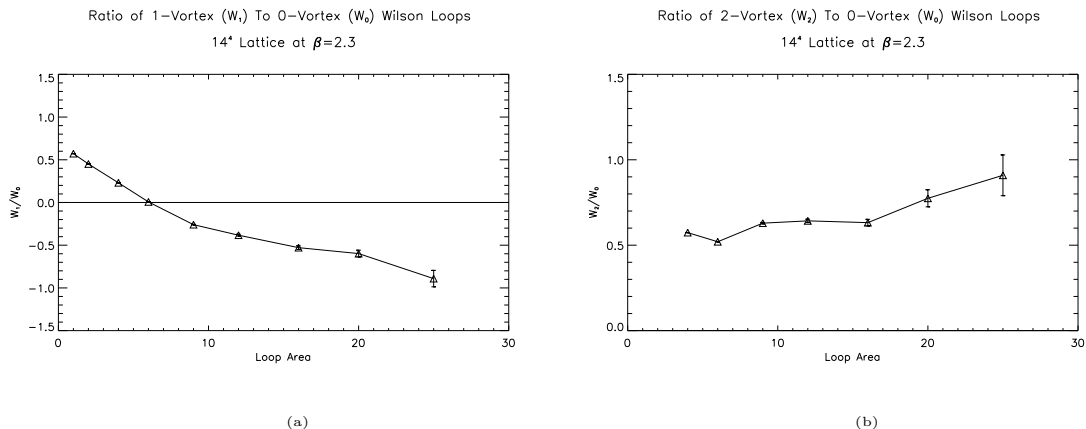
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Figure 3: (a) $W_1(C)/W_0(C)$, (b) $W_2(C)/W_0(C)$ vs. loop area at $\beta = 2.3$.

following arguments on measurements in center projected configurations. Instead, we will use center projection mainly for selecting sub-ensembles of configurations on which physical quantities are evaluated. In particular, we will compute $W_n(C)$, Wilson loops evaluated on such a sub-ensemble of configurations that precisely n P-vortices, in the corresponding center-projected configurations, pierce the minimal area of the loop. Though the data set is selected in center projection, the Wilson loops themselves are evaluated using the full, unprojected link variables.

From the computed vortex-limited Wilson loops $W_n(C)$ one can determine Creutz ratios χ_n. A simple test of a relation of P-vortices to confinement is then the following: if the presence/absence of P-vortices is *irrelevant* for confinement, then we would expect $\chi_0(I, J) \approx \chi(I, J)$ for large loops. The result of the test is shown in Figure 2a. The string tension vanishes if P-vortices are excluded from Wilson loops; it also vanishes if only odd numbers of P-vortices are excluded (Fig. 2b). The presence/absence of P-vortices seems strongly correlated with the presence/absence of "confiners" in unprojected field configurations.

However, the true "confiners" do not necessarily have to be any sort of Z_2 vortices. The natural question is, whether the objects identified using center projection tend to carry Z_2 magnetic flux. A simple argument [9, 10] leads to the expectation that $W_n(C)/W_0(C) \to (-1)^n$ for large loops.

Figure 3 shows our data for W_1/W_0 and W_2/W_0 at $\beta = 2.3$. They are consistent with the expectation and thus indicate that the confining gauge field configurations are center vortices. However, the values of $(-1)^n$ are reached for relatively *large* loop areas. The objects corresponding to P-vortices then appear to be rather "thick" Z_2 vortices. In Section 3.4 will this fact be related to a simple explanation of approximate Casimir scaling.

Now we are ready to formulate

Answer 1: We found evidence that center dominance in maximal center gauge is a reflection of the presence of thick center vortices in the unprojected configurations. Those vortices are identified as thin P-vortices in center projection.

3.2 Vortices and/or monopoles?

Question 2: If the vacuum is dominated, at long wavelengths, by Z_2 vortex configurations, then how do we explain the numerical successes of abelian projection in maximal abelian gauge?

We believe that the question is answered in the following way:

(Probable) Answer 2: A center vortex configuration, transformed to maximal abelian gauge and then abelian-projected, will appear as a chain of monopoles alternating with antimonopoles. These monopoles essentially arise because of the projection; they are condensed because the long vortices from which they emerge are condensed.
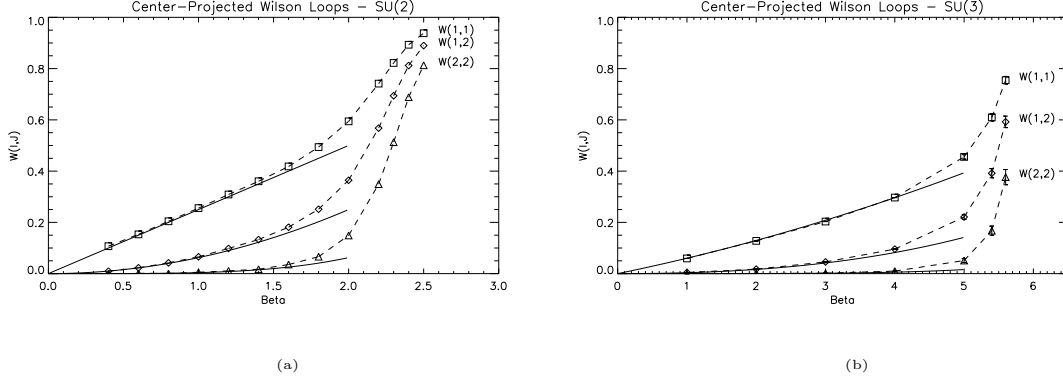
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Figure 4: Center-projected Wilson loops vs. the strong-coupling expansion (solid lines) in SU(2) and SU(3) lattice gauge theory. SU(3) values were obtained on an 8^4 lattice.

The support for the answer was given in much detail in [10]. It is clear, however, that this question deserves further study.

3.3 SU(3)?

Question 3: In nature quarks appear in three colours. Do the observed phenomena survive transition from SU(2) to SU(3)?

The maximal center gauge in SU(3) gauge theory is defined as the gauge which brings link variables U as close as possible to elements of its center $Z_3 = \{e^{-2i\pi/3}I, \ I, \ e^{2i\pi/3}I\}$. This can be achieved e.g. by maximizing the quantity

$$\sum_x \sum_\mu \mathrm{Re}\left([\mathrm{Tr}\ U_\mu(x)]^3\right). \tag{5}$$

Fixing to the maximal center gauge in SU(3) gauge theory turns out to be much more difficult and CPU-time consuming than in the case of SU(2). Therefore our simulations have until now been restricted to small lattice sizes and to strong coupling.

Our strong coupling results for the SU(2) and SU(3) lattice gauge theory are compared in Figure 4. In SU(2) Monte Carlo data agree with the strong coupling expansion up to almost $\beta = 1.5$. Figure 4b shows center-projected Wilson loops in SU(3) together with results of strong-coupling expansion to leading and next-to-leading order. The agreement extends up to $\beta = 4$.

(Partial) Answer 3: The situation at strong coupling looks much the same in SU(2) and SU(3): in both cases full Wilson loops are well reproduced by those constructed from center elements alone in MCG. Thus, center dominance is seen in SU(3) gauge theory at strong coupling.

3.4 Casimir scaling?

Question 4: Is there any way of accommodating the approximate Casimir scaling of higher-representation potentials to the vortex dominated QCD vacuum?

At the first sight, there is not. The adjoint representation transforms trivially under the gauge group center, large adjoint Wilson loops are unaffected by center vortices. As a result, the adjoint string tension vanishes.

There is however a loophole in the above statements: Adjoint loops are unaffected, unless the core of the vortex happens to overlap with the perimeter of the loop. If, then, the vortex thickness

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is quite large, on the order or exceeding the typical diameter of low-lying hadrons – and our data seem to indicate the presence of rather "thick" center vortices, – the Wilson loops can be influenced by vortices up to relatively large loop sizes.

A phenomenological model of the "thick" center vortex core has been worked out in a recent paper of three of us [7]. We cannot discuss the model in detail here, we just mention that it leads to potentials between colour sources that show approximate Casimir scaling at small and intermediate distances, and colour screening of integer-representation sources at large distances.

Answer 4: The Casimir scaling of the string tensions of higher-representation Wilson loops is an effect due to the finite (and large) thickness of the center vortex cores (see also [14]).

4   Conclusions

We subjected the picture of quark confinement based on center vortices to simple tests on the lattice. We proposed a method of localizing center vortices in thermalized lattice configurations, and found vortices to be responsible for the asymptotic string tension in SU(2) lattice gauge theory. The same holds also for SU(3) at strong coupling.

Further, we posed the lattice a lot of simple questions on the nature of vortex configurations. The tests indicate that the "confiners" in QCD are center vortices, and monopoles appear along vortices as artifacts of abelian projection. It is tempting to believe that monopole condensation might be just a manifestation of the underlying vortex condensation.

Finally, the vortices observed in our simulations possess a thick core; the thickness of the core is the cause for approximate Casimir scaling of potentials at intermediate distances. This solves a long-standing problem of the center vortex theory.

The "spaghetti vacuum" picture, believed to be dead for more than a decade, returns to the stage, in quite a good health.

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